

1D Global Bosonization of Quantum Gravity

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Abstract

Reduction of the Wheeler–DeWitt equation to the Klein–Gordon–Fock evolution for bosonic field by using of global bosonization to one-dimensional is proposed. The second quantization of the theory is carried out, and the Quantum Gravity is constructed in terms of the Fock–Bogoliubov–Heisenberg initial data operator basis. It is shown that this leads to understanding of mass of the bosonic field as a scaled initial data mass by one-point correlations of two bosonic fields.

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1 Introduction: Unsolved Quantum Gravity

The Einstein–Hilbert field equations of General Relativity [1, 2]¹

$$R_{\mu\nu} - \frac{R[g]}{2}g_{\mu\nu} + \Lambda g_{\mu\nu} = 3T_{\mu\nu}, \quad R[g] = g^{\kappa\lambda}R_{\kappa\lambda} \quad (1)$$

where $g_{\mu\nu}$ is a non-degenerate and symmetric $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ -tensor field, $R_{\mu\nu}, \Lambda, T_{\mu\nu}$ are the metric-contracted Riemann curvature tensor, cosmological constant, and stress-energy tensor, and $R[g]$ is the Ricci scalar curvature of a pseudo-Riemannian manifold (M, g) [3, 4], arise due to the Palatini principle [5]

$$\frac{\delta S[g]}{\delta g_{\mu\nu}} = 0, \quad (2)$$

used to the Einstein–Hilbert action modified by a boundary term

$$S[g] = -\frac{1}{3} \int_{\partial M} d^3x \sqrt{h} K[h] + \int_M d^4x \sqrt{-g} \left\{ -\frac{1}{6} R[g] + \frac{\Lambda}{3} + \mathcal{L} \right\}, \quad (3)$$

springs from allowing variations for which the normal derivatives on ∂M are non-zero, in order to cancel surface terms. Here $K[h]$ is the extrinsic curvature of an induced three-dimensional spacelike boundary $(\partial M, h)$, and \mathcal{L} is the Matter fields Lagrangian provoking the stress-energy tensor $T_{\mu\nu}$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}. \quad (4)$$

Stationarity of the Matter fields results in existence of a global timelike Killing vector field for a metric field $g_{\mu\nu}$. A coordinate system can be chose such that the Killing vector field equals $\frac{\partial}{\partial t}$ and the foliation $t = \text{constant}$ is spacelike. Then a metric field depends at most on a spatial coordinates x^i , so the t can be treated globally [6], and 3 + 1 decomposition of a metric

$$g_{\mu\nu} = \begin{bmatrix} -N^2 + N_i N^i & N_j \\ N_i & h_{ij} \end{bmatrix}, \quad g^{\mu\nu} = \begin{bmatrix} -\frac{1}{N^2} & \frac{N^j}{N^2} \\ \frac{N^i}{N^2} & h^{ij} - \frac{N^i N^j}{N^2} \end{bmatrix}, \quad (5)$$

$$h_{ik} h^{kj} = \delta_i^j, \quad N^i = h^{ij} N_j, \quad g = N^2 h, \quad (6)$$

¹We use the system of units $c = \hbar = k_B = 8\pi G/3 = 1$.

has also a global sense. In this case the action (3) becomes

$$S[g] = \int dt L(\pi, \pi^i, \pi^{ij}, N, N_i, h_{ij}), \quad (7)$$

$$L(\pi, \pi^i, \pi^{ij}, N, N_i, h_{ij}) = \int_{\partial M} d^3x \left\{ \pi \dot{N} + \pi^i \dot{N}_i + \pi^{ij} \dot{h}_{ij} - NH - N_i H^i \right\} \quad (8)$$

where

$$\dot{h}_{ij} = \frac{\partial h_{ij}}{\partial t} = N_{i|j} + N_{j|i} - 2NK_{ij}, \quad (9)$$

$$H = \sqrt{h} \{ K^2 - K_{ij} K^{ij} + R[h] - 2\Lambda - 6T_{nn} \}, \quad (10)$$

$$H^i = -2\pi^{ij}_{;j} = -2\pi^{ij}_{,j} - h^{il} (2h_{jl,k} - h_{jk,l}) \pi^{jk}, \quad (11)$$

where the second formula follows from the Gauss-Codazzi equations [7]. Here K_{ij} is the extrinsic-curvature tensor ($K = K^i_i$), and π^{ij} is the canonical conjugate momentum field to the field h_{ij}

$$\pi^{ij} = \frac{\delta L}{\delta \dot{h}_{ij}} = -\sqrt{h} (K^{ij} - h^{ij} K). \quad (12)$$

Time-preservation requirement [8] of the primary constraints [9] for (7)

$$\pi = \frac{\delta L}{\delta \dot{N}} \approx 0, \quad \pi^i = \frac{\delta L}{\delta \dot{N}_i} \approx 0, \quad (13)$$

leads to the secondary constraints

$$H \approx 0, \quad H^i \approx 0, \quad (14)$$

called the Hamiltonian constraint and the diffeomorphism constraint, respectively. The diffeomorphism constraint merely reflects spatial diffeoinvariance, and the Hamiltonian constraint gives the dynamics. By (12) the Hamiltonian constraint becomes the Einstein–Hamilton–Jacobi equation [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44]

$$G_{ijkl} \pi^{ij} \pi^{kl} + \sqrt{h} (R[h] - 2\Lambda - 6T_{nn}) = 0, \quad (15)$$

where G_{ijkl} is called the Wheeler superspace metric

$$G_{ijkl} = \frac{1}{2\sqrt{h}} (h_{ik} h_{jl} + h_{il} h_{jk} - h_{ij} h_{kl}). \quad (16)$$

Canonical quantization [45] of (15) by the commutation relations [46]

$$i [\pi^{ij}(x), h_{kl}(y)] = \frac{1}{2} (\delta_k^i \delta_l^j + \delta_l^i \delta_k^j) \delta^{(3)}(x, y), \quad (17)$$

$$i [\pi^i(x), N_j(y)] = \delta_j^i \delta^{(3)}(x, y), \quad (18)$$

$$i [\pi(x), N(y)] = \delta^{(3)}(x, y), \quad (19)$$

leads to the Wheeler–DeWitt equation [47, 9]

$$\left\{ -G_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} + h^{1/2} (R[h] - 2\Lambda - 6T_{nn}) \right\} \Psi[h] = 0, \quad (20)$$

and the other first class constraints are conditions on the wave function $\Psi[h]$

$$\pi \Psi[h] = 0, \quad \pi^i \Psi[h] = 0, \quad H^i \Psi[h] = 0. \quad (21)$$

Furthermore, the canonical commutation relations hold

$$[\pi(x), \pi^i(y)] = [\pi(x), H^i(y)] = [\pi^i(x), H^j(y)] = [\pi^i(x), H(y)] = 0, \quad (22)$$

and in consequence H_i are generators of diffeomorphisms $\tilde{x}^i = x^i + \delta x^i$ [9]

$$\left[h_{ij}, i \int_{\partial M} H_a \delta x^a d^3 x \right] = -h_{ij,k} \delta x^k - h_{kj} \delta x^k_{,i} - h_{ik} \delta x^k_{,j}, \quad (23)$$

$$\left[\pi_{ij}, i \int_{\partial M} H_a \delta x^a d^3 x \right] = -(\pi_{ij} \delta x^k)_{,k} + \pi_{kj} \delta x^i_{,k} + \pi_{ik} \delta x^j_{,k}, \quad (24)$$

or in more conventional form

$$i [H_i(x), H_j(y)] = \int_{\partial M} H_a c_{ij}^a d^3 z, \quad (25)$$

$$i [H(x), H_i(y)] = H \delta_{,i}^{(3)}(x, y), \quad (26)$$

$$i \left[\int_{\partial M} H \delta x_1 d^3 x, \int_{\partial M} H \delta x_2 d^3 x \right] = \int_{\partial M} H^a (\delta x_{1,a} \delta x_2 - \delta x_1 \delta x_{2,a}) d^3 x, \quad (27)$$

where $H_i = h_{ij} H^j$, and

$$c_{ij}^a = \delta_i^a \delta_j^b \delta_{,b}^{(3)}(x, z) \delta^{(3)}(y, z) - \delta_j^a \delta_i^b \delta_{,b}^{(3)}(y, z) \delta^{(3)}(x, z), \quad (28)$$

are structure constants of diffeomorphism group. Commutators (25–27) show the first-class constrained system property.

The Wheeler–DeWitt equation (20) has been studied intensively since 30 years [48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65,

66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77]. In fact, this is an equation on superspace [78], defined as a space of all equivalence class of metrics related by the action of the diffeomorphism group of a compact, connected, orientable, Hausdorff, C^∞ 3-dimensional spacelike manifold without boundary ∂M . If $Riem(\partial M)$ consists all C^∞ Riemannian metrics on ∂M , and $Diff(\partial M)$ is a group of all C^∞ diffeomorphisms of ∂M that preserve orientation, then the Wheeler superspace $S(\partial M)$ is the space of all orbits of $Diff(\partial M)$, *i.e.* $S(\partial M) = Riem(\partial M)/Diff(\partial M)$. $S(\partial M)$ is a connected, second-countable, metrizable space. All geometries with the same kind of symmetry are manifold in $S(\partial M)$ - they have homeomorphic neighbourhoods. However, symmetric geometries neighbourhoods are not homeomorphic to nonsymmetric geometries ones, and by this $S(\partial M)$ is not manifold. Superspace can be decomposed by its subspaces on a countable, partially-ordered, C^∞ -Fréchet manifold partition, that is an inverted stratification indexed by the symmetry type - geometries with a given symmetry are completely contained within the boundary of less symmetric geometries. The minisuperspace models, *i.e.* Quantum Cosmology [79, 80, 81, 82, 83, 84, 85], study certain strata of superspace. Fischer [78] proved by suitable choice of a subgroup of $Diff(\partial M)$ and by action of this subgroup on $Riem(\partial M)$, for n -dimensional ∂M the superspace $S(\partial M)$ can be extended to a manifold $S_e(\partial M)$ such that $\dim S_e(\partial M)/S(\partial M) = n(n+1)$.

2 Global Bosonization to One Dimension

The superspace has no physical consequences [86] and is the main structural problem of the theory. In this section we will construct linearization of the Quantum Gravity, global bosonization to one dimension.

2.1 Reduction Problem

Let us consider the standard relation of General Relativity [87] between variations of a metric field determinant and a metric field

$$\delta g = g g^{\mu\nu} \delta g_{\mu\nu} = g (g^{00} \delta g_{00} + g^{ij} \delta g_{ij} + g^{0j} \delta g_{0j} + g^{i0} \delta g_{i0}). \quad (29)$$

The 3 + 1 parametrization (5) allows determine the partial variations

$$\delta g_{00} = -\delta N^2 + N^i N^j \delta h_{ij} + h_{ij} N^i \delta N^j + h_{ij} N^j \delta N^i, \quad (30)$$

$$\delta g_{ij} = \delta h_{ij}, \quad (31)$$

$$\delta g_{0j} = h_{ij} \delta N^i + N^i \delta h_{ij}, \quad (32)$$

$$\delta g_{i0} = h_{ij} \delta N^j + N^j \delta h_{ij}, \quad (33)$$

as well as the total variation

$$\delta g = N^2 \delta h + h \delta N^2. \quad (34)$$

Taking a contravariant metric field components of (5) we obtain from (29)

$$N^2 \delta h = N^2 h h^{ij} \delta h_{ij}, \quad (35)$$

so that the global relation between first functional derivatives is established

$$\frac{\delta}{\delta h_{ij}} = h h^{ij} \frac{\delta}{\delta h}. \quad (36)$$

The global reduction (36) has deep sense - the first functional derivative operator $\frac{\delta}{\delta h_{ij}}$ is an object from a vector space spanned by the contravariant 3-space metric h^{ij} . Therefore, as the consequence of (36) one can determine the Wheeler–DeWitt second derivative functional operator (20)

$$-G_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} = \frac{3}{2} h^{3/2} \frac{\delta^2}{\delta h^2}, \quad (37)$$

where was used the obvious identity

$$(h_{ik} h_{jl} + h_{il} h_{jk} - h_{ij} h_{kl}) h^{ij} h^{kl} = \delta_i^l \delta_l^i + \delta_l^j \delta_j^l - \delta_i^i \delta_k^k = -3. \quad (38)$$

Hence the Wheeler–DeWitt equation (20) becomes the one-dimensional Klein–Gordon–Fock type evolution

$$\left(\frac{\delta^2}{\delta h^2} + m^2 \right) \Psi[h] = 0, \quad (39)$$

where

$$m^2 \equiv m^2[h] = \frac{2}{3h} (R[h] - 2\Lambda - 6T_{nn}), \quad (40)$$

is the square of mass of the bosonic field $\Psi[h]$. By using of the notation

$$\Phi = \begin{bmatrix} \Psi \\ \Pi_\Psi \end{bmatrix}, \quad \vec{\partial} = \begin{bmatrix} \delta \\ \frac{\delta}{\delta h} \end{bmatrix}, \quad \mathbb{M} = \begin{bmatrix} 0 & 1 \\ -m^2 & 0 \end{bmatrix} \geq 0, \quad (41)$$

the second order scalar equation (39) becomes the first order vector equation

$$(i\Gamma \vec{\partial} - \mathbb{M}) \Phi[h] = 0, \quad (42)$$

where Γ matrices obey the relations

$$\Gamma = [-i\mathbf{1}, \mathbf{0}], \quad \{\Gamma^a, \Gamma^b\} = 2\eta^{ab}\mathbf{1}, \quad \eta^{ab} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \quad (43)$$

where $\mathbf{1}$ and $\mathbf{0}$ are unit and null two-dimensional matrices.

We have seen that application of the global reduction (36) to the Wheeler–DeWitt equation (20), that has also a global nature by a character of the decomposition (5), results in the bosonic quantum mechanics (39). This scalar-type second order functional evolution was reduced directly to the vector-type first order functional equation (42) with some two-component field $\Phi[h]$ as a solution. In the equation (39) as well as in its the reduced form (42) the superspace metric is completely absent. By this reason the most mysterious element of the Wheeler Quantum Gravity’s logics was formally excluded from considerations – the notion of superspace as well as its mathematical properties are not need to further analysis. In further developments of this paper we will concentrate on canonical quantization in the bosonic Fock space of the reduced equation (42).

2.2 Fock–Bogoliubov–Heisenberg initial data basis

Next step of the bosonization is the field quantization of the equation (42)

$$\Phi[h] \rightarrow \mathbf{\Phi}[h] \Rightarrow \left(i\Gamma\vec{\partial} - \mathbb{M}\right) \mathbf{\Phi}[h] = 0, \quad (44)$$

according to canonical commutation relations proper for the Bose statistics [88, 89, 90]

$$[\mathbf{\Pi}_\Psi[h'], \mathbf{\Psi}[h]] = -i\delta(h' - h), \quad (45)$$

$$[\mathbf{\Pi}_\Psi[h'], \mathbf{\Pi}_\Psi[h]] = 0, \quad (46)$$

$$[\mathbf{\Psi}[h'], \mathbf{\Psi}[h]] = 0. \quad (47)$$

By using of the second quantization method [91, 92, 93], from the equation (39) spring that the field operator $\mathbf{\Phi}[h]$ of the reduced equation (42) can be represent in the Fock space of annihilation and creation functional operators

$$\mathbf{\Phi}[h] = \mathbb{Q}[h]\mathfrak{B}[h], \quad (48)$$

where $\mathfrak{B}[h]$ is a dynamical basis in the Fock space

$$\mathfrak{B}[h] = \left\{ \begin{bmatrix} G[h] \\ G^\dagger[h] \end{bmatrix} : [G[h'], G^\dagger[h]] = \delta(h' - h), [G[h'], G[h]] = 0 \right\}, \quad (49)$$

and $\mathbb{Q}[h]$ is the second quantization matrix

$$\mathbb{Q}[h] = \begin{bmatrix} \frac{1}{\sqrt{2|m[h]|}} & \frac{1}{\sqrt{2|m[h]|}} \\ -i\sqrt{\frac{|m[h]|}{2}} & i\sqrt{\frac{|m[h]|}{2}} \end{bmatrix}. \quad (50)$$

In this way the operator equation (44) becomes the equation for a basis $\mathfrak{B}[h]$

$$\frac{\delta \mathfrak{B}[h]}{\delta h} = \begin{bmatrix} -im[h] & \frac{1}{2m[h]} \frac{\delta m[h]}{\delta h} \\ \frac{1}{2m[h]} \frac{\delta m[h]}{\delta h} & im[h] \end{bmatrix} \mathfrak{B}[h]. \quad (51)$$

Actually, there is a nonlinearity given by coupling between annihilation and creation operators present as nondiagonal terms in (51), so the equation (51) can not be solved standardly. In order to solving, let us suppose that in the Fock space exists a new basis $\mathfrak{B}'[h]$

$$\mathfrak{B}'[h] = \left\{ \begin{bmatrix} G'[h] \\ G'^{\dagger}[h] \end{bmatrix} : [G'[h'], G'^{\dagger}[h]] = \delta(h' - h), [G'[h'], G'[h]] = 0 \right\}, \quad (52)$$

for which the the Bogoliubov transformation

$$\mathfrak{B}'[h] = \begin{bmatrix} u[h] & v[h] \\ v^*[h] & u^*[h] \end{bmatrix} \mathfrak{B}[h], \quad |u[h]|^2 - |v[h]|^2 = 1, \quad (53)$$

and the Heisenberg evolution

$$\frac{\delta \mathfrak{B}'[h]}{\delta h} = \begin{bmatrix} -i\lambda[h] & 0 \\ 0 & i\lambda[h] \end{bmatrix} \mathfrak{B}'[h], \quad (54)$$

are supposed to hold together. We will call briefly this special basis as the Fock–Bogoliubov–Heisenberg (FBH) operator basis. The diagonalization procedure (52)-(54) converts the operator basis evolution (51) onto the Bogoliubov coefficients one

$$\frac{\delta}{\delta h} \begin{bmatrix} u[h] \\ v[h] \end{bmatrix} = \begin{bmatrix} -im[h] & \frac{1}{2m[h]} \frac{\delta m[h]}{\delta h} \\ \frac{1}{2m[h]} \frac{\delta m[h]}{\delta h} & im[h] \end{bmatrix} \begin{bmatrix} u[h] \\ v[h] \end{bmatrix}, \quad (55)$$

and the basis $\mathfrak{B}'[h]$ takes a meaning of static operator basis associated with initial data

$$\mathfrak{B}'[h] \equiv \mathfrak{B}_I = \left\{ \begin{bmatrix} G_I \\ G_I^{\dagger} \end{bmatrix} : [G_I, G_I^{\dagger}] = 1, [G_I, G_I] = 0 \right\}, \quad (56)$$

within the vacuum state can be correctly defined

$$|0\rangle_I = \left\{ |0\rangle_I : \mathbb{G}_I |0\rangle_I = 0, \ 0 = {}_I\langle 0 | \mathbb{G}_I^\dagger \right\}. \quad (57)$$

In the other words, the integrability problem consists in the equations (55). However, the Bogoliubov coefficients are additionally constrained by the hyperbolic rotation condition (52). By this it is useful to apply the superfluid parametrization, for which the solutions are

$$u[h] = \frac{1 + \mu[h]}{2\sqrt{\mu[h]}} \exp \left\{ im_I \int_{h_I}^h \frac{\delta h'}{\mu[h']} \right\}, \quad (58)$$

$$v[h] = \frac{1 - \mu[h]}{2\sqrt{\mu[h]}} \exp \left\{ -im_I \int_{h_I}^h \frac{\delta h'}{\mu[h']} \right\}, \quad (59)$$

where $\mu[h]$ is a mass scale

$$\mu[h] = \frac{m_I}{m[h]}. \quad (60)$$

This establish the relation between a dynamical basis $\mathfrak{B}[h]$ and the initial data FBH basis \mathfrak{B}_I as follows

$$\mathfrak{B}[h] = \mathbb{G}[h] \mathfrak{B}_I, \quad (61)$$

where the transformation matrix $\mathbb{G}[h]$ is

$$\mathbb{G}[h] = \begin{bmatrix} \frac{\mu[h] + 1}{2\sqrt{\mu[h]}} e^{-i\theta[h]} & \frac{\mu[h] - 1}{2\sqrt{\mu[h]}} e^{i\theta[h]} \\ \frac{\mu[h] - 1}{2\sqrt{\mu[h]}} e^{-i\theta[h]} & \frac{\mu[h] + 1}{2\sqrt{\mu[h]}} e^{i\theta[h]} \end{bmatrix}, \quad (62)$$

where $i\theta[h]$ is given by a phase of (58). By this reason, the solution of the equation (44) can be expressed in the initial data basis as

$$\Phi[h] = \mathbb{Q}[h] \mathbb{G}[h] \mathfrak{B}_I. \quad (63)$$

2.3 One-point correlations

The second quantized equation (39), *i.e.*

$$\left(\mu^2[h] \frac{\delta^2}{\delta h^2} + m_I^2 \right) \Psi[h] = 0, \quad (64)$$

has a solution

$$\Psi[h] = \frac{\mu[h]}{2\sqrt{2m_I}} \left(\exp \left\{ -im_I \int_{h_I}^h \frac{\delta h'}{\mu[h']} \right\} G_I + \exp \left\{ im_I \int_{h_I}^h \frac{\delta h'}{\mu[h']} \right\} G_I^\dagger \right), \quad (65)$$

that is a direct conclusion of the relation (63). This field acts on the initial data vacuum state as follows

$$\Psi[h]|0\rangle_I = \frac{\mu[h]}{2\sqrt{2m_I}} e^{i\theta[h]} G_I^\dagger |0\rangle_I, \quad (66)$$

$${}_I\langle 0|\Psi^\dagger[h] = {}_I\langle 0|G_I \frac{\mu[h]}{2\sqrt{2m_I}} e^{-i\theta[h]}. \quad (67)$$

By this reason, one can consider the following many-field quantum states

$$|h, n\rangle \equiv (\Psi[h])^n |0\rangle_I = \left(\frac{\mu[h]}{2\sqrt{2m_I}} e^{i\theta[h]} \right)^n G_I^{\dagger n} |0\rangle_I, \quad (68)$$

$$\langle n', h'| \equiv {}_I\langle 0|(\Psi^\dagger[h'])^{n'} = {}_I\langle 0|G_I^{n'} \left(\frac{\mu[h']}{2\sqrt{2m_I}} e^{-i\theta[h']} \right)^{n'}, \quad (69)$$

and determine the two-point quantum correlator of two many-field states

$$\langle n', h'|h, n\rangle = \frac{\mu^{n'}[h']\mu^n[h]}{(8m_I)^{(n'+n)/2}} e^{-im_I\theta_{n',n}[h',h]} \langle 0|G_I^{n'} G_I^{\dagger n}|0\rangle_I, \quad (70)$$

where

$$\theta_{n',n}[h',h] = n' \int_{h_I}^{h'} \frac{\delta h''}{\mu[h'']} - n \int_{h_I}^h \frac{\delta h''}{\mu[h'']}. \quad (71)$$

Application of the normalization

$$\langle 1, h_I|h_I, 1\rangle = \frac{1}{8m_I} {}_I\langle 0|0\rangle_I \equiv 1 \implies {}_I\langle 0|0\rangle_I = 8m_I, \quad (72)$$

allows define the following correlators

$$\langle n', h|h, n\rangle = \left(\frac{\langle 1, h|h, 1\rangle}{{}_I\langle 0|0\rangle_I} \right)^{(n'+n)/2} e^{-i(n'-n)\theta[h]} {}_I\langle 0|G_I^{n'} G_I^{\dagger n}|0\rangle_I, \quad (73)$$

$$\frac{\langle n, h'|h, n\rangle}{{}_I\langle 0|0\rangle_I} = \left(\frac{\langle 1, h'|h, 1\rangle}{{}_I\langle 0|0\rangle_I} \right)^n, \quad (74)$$

where

$$\langle 1, h'|h, 1\rangle = \mu[h']\mu[h] \exp \left\{ im_I \int_{h'}^h \frac{\delta h''}{\mu[h'']} \right\}, \quad (75)$$

$$\langle 1, h|h, 1\rangle = \mu^2[h]. \quad (76)$$

The last formula (76) together with the definition (60) leads to the relation between the mass of the bosonic field $\Psi[h]$ and the initial data mass m_I

$$m[h] = \lambda[h]m_I, \quad \lambda[h] = \frac{1}{\sqrt{\langle 1, h|h, 1 \rangle}}, \quad (77)$$

that means the arbitrary mass $m[h]$ is only rescaled the initial data mass m_I , and the scale λ is directly related to one-point correlations of the quantum bosonic field $\Psi[h]$. Therefore, actually the mass $m[h]$ for arbitrary h is given by correlations of two bosonic fields Ψ in the point h . Finally note that the two-point correlator (75), that can be rewritten in the power series form

$$\langle 1, h'|h, 1 \rangle = \mu[h']\mu[h] \prod_{n=0}^{\infty} \sum_{p=0}^{\infty} a_{pn}[h, h'|h_I] \left(\frac{\delta^n}{\delta h^n} \mu^2[h] \Big|_{h_I} \right)^p, \quad (78)$$

with a coefficients

$$a_{pn}[h, h'|h_I] = \frac{1}{p!} \left[im_I \frac{(2n-3)!}{2^{2n-1}(n-1)!} \sum_{k=0}^{n+1} \frac{(-1)^k}{k!(n-k+1)!} (h_I)^{n-k+1} (h^k - h'^k) \right]^p. \quad (79)$$

The series gives an opportunity to study perturbationally the two-point correlations around the initial data point $h = h_I$.

3 Summary

In spite of a work in the Hamiltonian approach to General Relativity and the primary quantization, the method of global bosonization to one h -dimension of the Wheeler–DeWitt Quantum Gravity and its second quantization in the Fock–Bogoliubov–Heisenberg initial data basis, which was presented in details in this paper differs seriously from the previous authors considerations. The main difference is a quantum field theory formulation of the Quantum Gravity, that leads to the FBH initial data basis and considering the theory in terms of the quantum bosonic field $\Psi[h]$ associated with a 3-dimensional induced spacelike geometry $(\partial M, h)$. The proposed approach is not the so called third quantization [94, 95, 96, 97, 98, 99, 100], where the Fock operator bases formalism is not applied. The main goal of the presented linearization is a canceling of the Wheeler’s superspace notion from considerations, and formulation of the Quantum Gravity in terms of the Klein–Gordon–Fock operator evolution and the one-point correlations, that results in the mass scale of the system.

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